THE LAST PUSH

MATHEMATICS

PAPER 1

GRADE 12

2017

FORMULA SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad$ $A = P(1 - ni)$ $\quad$ $A = P(1 - i)^n$ $\quad$ $A = P(1 + i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a(r^n - 1)}{r - 1}\ ; r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}\ ;\ -1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad$ $P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In $\triangle ABC$: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \triangle ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha\sin\beta$

$\cos 2\alpha = cos^2\alpha - sin^2\alpha$

$= 1 - 2sin^2\alpha$ $\quad$ $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$= 2cos^2\alpha - 1$

$\bar{x} = \frac{\sum fx}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_1 - x)^2}{\text{n}}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(A\ of\ B) = P(A) + P(B) - P(A\ en\ B)$

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

QUESTION 1

Solve for x:

1.1 $2x(x+1)-7(x+1)=0$ (3)

1.2 $x^2-5x-1=0$, correct to two decimal places. (3)

1.3 $4x^2+1\geq 5x$ (4)

1.4 $5^{4x+3}.100^{-2x+1}=50\ 000$ (4)

1.5 Solve for x and y simultaneously.

$$x=2y \quad \text{and} \quad x^2+2x-y-y^2=36$$ (5)

1.6 Show that the roots of $x^2-kx+k-1=0$ are real and rational for all real values of $\boldsymbol{k}$. (4)

[23]

QUESTION 2

2.1 Given the following linear series: $3\ +\ 7\ +\ 11\ +\ \ldots\ \ldots\ \ldots\ +\ 483$

2.1.1 How many terms does the above series have? (3)

2.1.2 Write the above series in sigma notation. (2)

2.2 $2t-4\ ;\ t-3\ ;\ 8-2t$ are the 10^{th}, 11^{th} and 12^{th} terms of an arithmetic sequence.

2.2.1 Determine the value of $\boldsymbol{t}$. (3)

2.2.2 Calculate the value of the first term. (3)

2.3 The following information of a geometric pattern is given:
$T_1+T_2=-1$ and $T_3+T_4=-4$

Determine the numerical values of the first three terms if $r>0$. (6)

[17]

QUESTION 1

1.1 Solve for x:

1.1.1. $x^2 = 8x$ (3)

1.1.2. $4x^2 - 4x - 4 = 0$ (correct to two decimal places) (3)

1.1.3. $(2-x)(x+4) \leq 0$ (3)

1.1.4. $2.3^{x+1} - 3.3^{x-1} = 104$ (4)

1.2 If $\sqrt{10} = 3{,}162$, calculate without the use of a calculator, the value of $\sqrt[5]{316{,}2}$ (4)

1.3 Solve for x and y simultaneously:

$x = y + 1$ and $0 = (2x-1)(x+2y-3)$ (6)

[23]

QUESTION 2

2.1 Given the quadratic sequence : $6\ ;\ x\ ;\ 26\ ;\ 45\ ;\ y$

2.1.1 Determine the first four first differences in terms of x and y (2)

2.1.2 Calculate the values of x and y. (4)

2.2 Calculate the value of n, if:

$$\sum_{k=1}^{n}(4-3k) = -125$$ (5)

[11]

QUESTION 3

3.1 The n-th term of a geometeric series is $T_n = x(x+1)^{n-1}$

3.1.1 Determine the constant ratio, in terms of x, in its simplest form. (2)

3.1.2 Determine the values of x so that the series $\sum_{n=1}^{\infty} x(x+1)^{n-1}$ converges. (3)

3.2 Calculate S_∞ (3)

3.3 If $x = 1$, write down the first three terms of the geometric series (2)

3.4 Determine the sum of the first 25 terms of the series calculated in Question 3.3 (3)

[13]

QUESTION 1

1.1 Solve for x :

1.1.1 $4x - x^3 = 0$ (3)

1.1.2 $\sqrt{4x+3} = x$ (4)

1.1.3 $(2.2^x - 1)(2^x + 2) = 0$ (3)

1.2 Given : $f(x) = x^2 + x - 11$

Solve for x in each of the following:

1.2.1 $f(x) = 0$ (3)

1.2.2 $f(x) = f'(x)$ (4)

1.3 Solve for x and y if :

$-x^2 + 4x + 5 - y = 0 \quad and \quad y - 8x - 9 = 0$ (6)

1.4 Consider the quadratic equation $ax^2 + bx + c = 0$.

1.4.1 Discuss the nature of the roots if $b^2 - 4ac > 0$. (2)

1.4.2 The roots is given by:

$$x = \frac{-3 \pm \sqrt{(k-1)(2-k)}}{2}.$$

Determine the value of k if the roots are non-real. (3)

[28]

QUESTION 1

1.1 Solve for x:

1.1.1 $9x^2 - 25 = 0$ (2)

1.1.2 $(2x-1)(x-2) = 4$ (Correct to ONE decimal place) (4)

1.1.3 $2x - 3\sqrt{x} - 2 = 0$ (4)

1.1.4 $-3(x-2)(x+4) > 0$ (4)

1.2 Consider $27^{\frac{x}{3}} = 3^{y-1}$ and $2x^2 - y = 5$

1.2.1 Show that $x = y - 1$ (2)

1.2.2 Solve for x and y simultaneously (5)

1.3 Determine for which values of m the roots of $mx^2 - 3x + 2 = x^2$ will be real and unequal. (5)

[26]

QUESTION 2

2.1 Which term in this sequence 36; 25; 14; . . . is equal to -52? (3)

2.2 Determine:

$$\sum_{k=1}^{8} \frac{1}{4}(2)^{k-1}$$ (3)

2.3 In a quadratic pattern with $T_n = an^2 + bn + c$, the second term is equal to 8 and the first differences of the quadratic sequence is given as: $6; 12; 18; \ldots\ldots\ldots\ldots$

2.3.1 Write down the values of the first four terms of the quadratic sequence. (3)

2.3.2 Calculate the value of T_{40} of the quadratic sequence. (5)

[14]

QUESTION 3

3.1 Consider the geometric series: $(x^2 - 9) + (x + 3) + (-2) + \ldots\ldots\ldots$

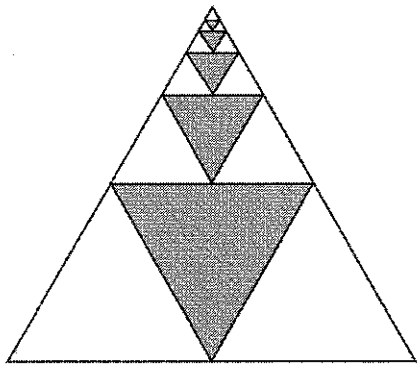
3.1.1 Show that $x = 1$. Show all calculations. (3)

3.1.2 Explain why the series will converge. (2)

3.1.3 Calculate the value of n if : $2\,S_n = -10$. (4)

3.2 Refer to the diagram alongside:

The largest triangle has an area of one square unit.
The biggest grey triangle has area $\frac{1}{4}$ sq. units and each subsequent grey triangle's area is $\frac{1}{4}$ the size of the triangle bigger than it. These triangles continue indefinitely.



Determine the area of the unshaded part of the diagram. (4)

[13]

QUESTION 2

2.1 The following series is a combination of an arithmetic and geometric series:

$$6+4+3+2+\frac{3}{2}+0+\frac{3}{4}+....$$

2.1.1 Write down the next two terms of the series. (2)

2.1.2 Find the sum of the first 113 terms of the series. (5)

2.1.3 Give a reason why the geometric series converges. (1)

2.2 In an arithmetic sequence, the first and the seventh terms are x^2 and $\left(x+6-5x^2\right)$ respectively.

If the common difference is x, determine the possible values of x. (4)

2.3 Consider: $0\ ;\ -1\ ;\ 1\ ;\ x\ ;\ 14$

2.3.1 Solve for x if the terms form a quadratic sequence. (3)

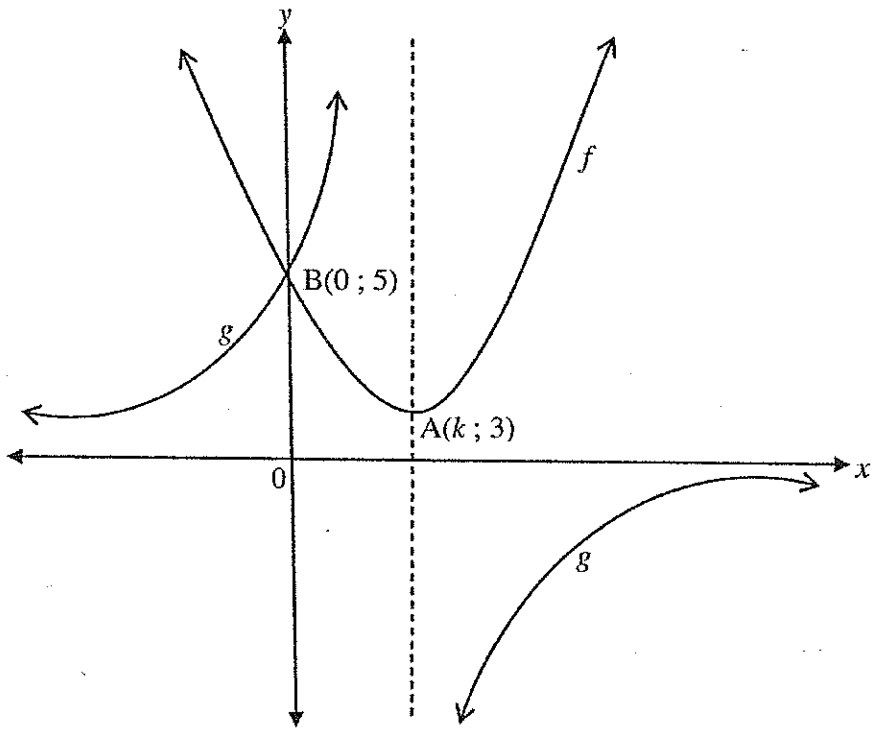
2.3.2 Calculate the general ter m of the sequence.. (4)

2.4 Calculate p if $\sum_{n=1}^{\infty} 27p^k = \sum_{t=2}^{5}\left(1-t^2\right)$ (6)

[25]

QUESTION 3

In the diagram, $f(x) = ax^2 + bx + c$ and $g(x) = \dfrac{-2}{x-2}$ are drawn. $B(0\ ;\ 5)$ is the y - intercept of f and g. The turning point, $A(k\ ;\ 3)$, of f is on the vertical asymptote of g.



3.1 Determine the equations of the asymptotes of g. (2)

3.2 Determine the equation of f in the form $y = ax^2 + bx + c$ (4)

3.3 Use the graph to determine the value(s) of k for which the graph of f has real and positive roots. (2)

3.4 Determine the turning point of p if $p(x) = -f(x+4)$. (2)

3.5 Determine the domain of the reflection of g over the y - axis. (2)

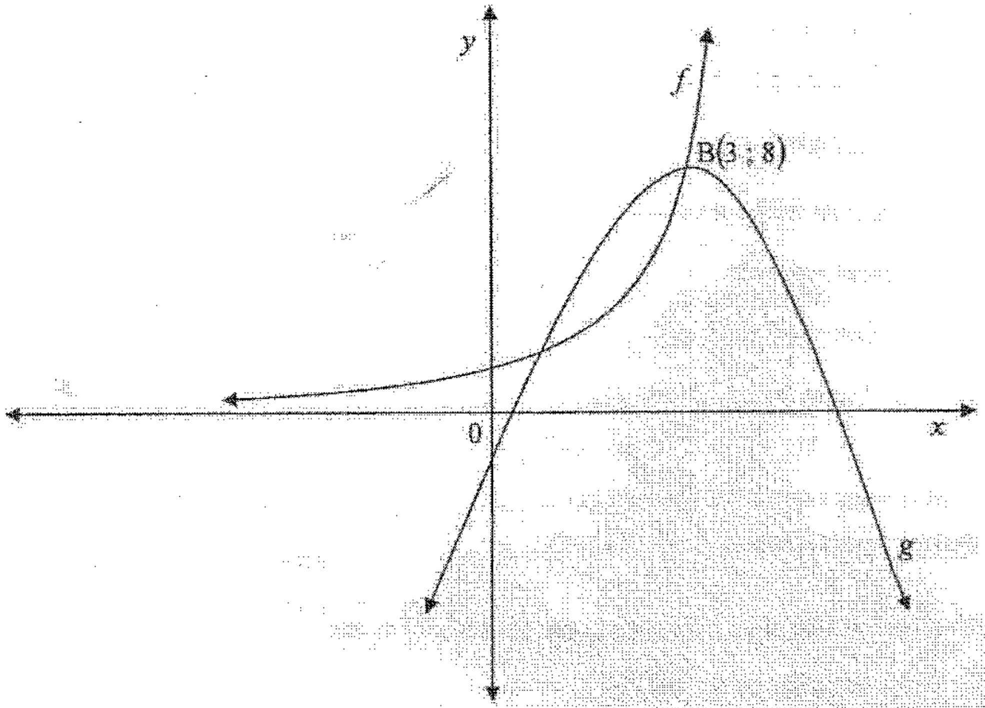
3.6 Determine for which value of x will $y = 5x + t$ be a tangent to the graph of f. (3)

[15]

QUESTION 4

Sketched below are the graphs of $f(x) = 2^x$ and $g(x) = -(x-p)^2 + q$, where q is constant.

B (3 ; 8) is one of the points of intersection of f and g. B is also the turning point of g.



4.1 Determine the equation of g in the form $y = ax^2 + bx + c$ (3)

4.2 Determine the range of $g(x) - 2$. (2)

4.3 For which values of k will $g(x) - k = 0$, has non-real roots? (2)

4.4 Write down the equation of $f^{-1}(x)$ in the form $y = \ldots\ldots$ (2)

4.5 Sketch the graph of f^{-1}. Indicate the x-intercepts and the coordinates of one other point on your graph. (3)

4.6 For which values of x will $f^{-1}(x) \leq 3$? (2)

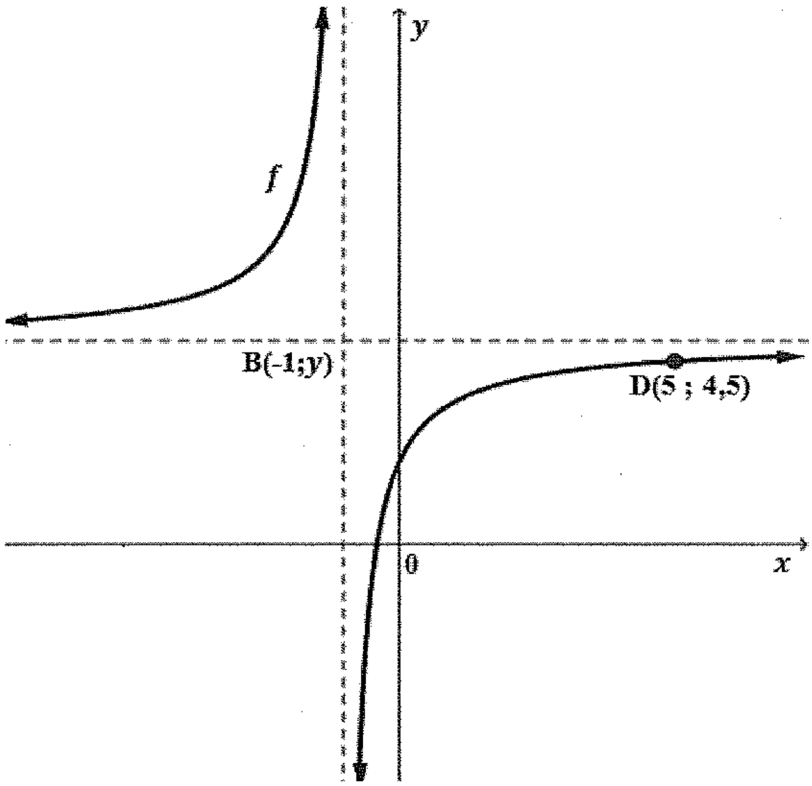
4.7 Determine for which values of x will $\dfrac{f'(x)}{g'(x)} > 0$ (2)

[16]

QUESTION 4

Sketched below is the graphs of $f(x)=\dfrac{-3}{x+p}+q$. The asymptotes of f intersect at B(-1; y).

D(5; 4,5) is a point on f.



4.1 Write down the value of p. (1)

4.2 Prove that the value of $q = 5$. (2)

4.3 Calculate the x-intercept of f. (2)

4.4 Write down the equation of the vertical asymptote of h if $h(x) = f(x + 4)$. (2)

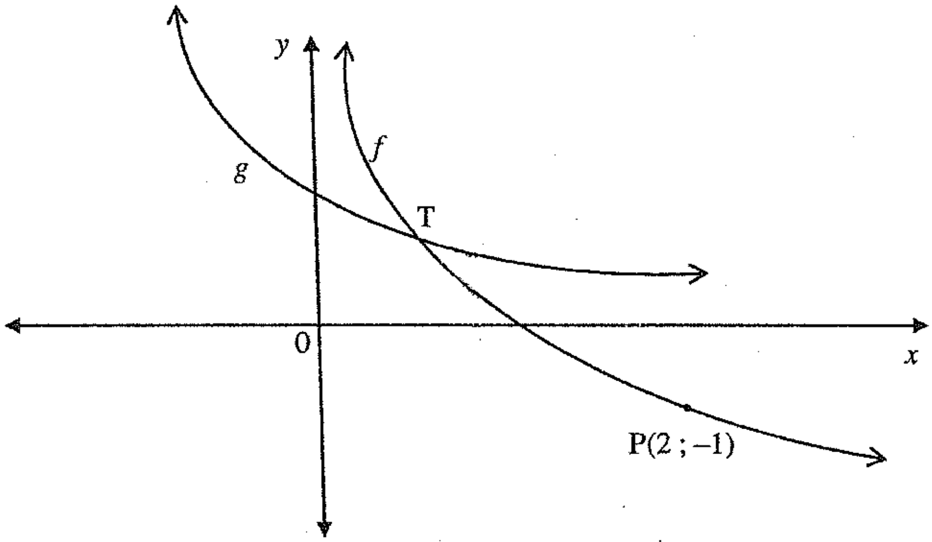
4.5 One of the axes of symmetry of f is an increasing function. Determine the equation of this axis of symmetry. (3)

4.6 Determine the values of x for which $f(x) \geq x+6$. (2)

[12]

QUESTION 4

In the diagram, the graphs of $f(x) = \log_a x$ and $g(x) = f^{-1}(x)$ are drawn. P $(2\,;-1)$ is a point on f. The two graphs intersect at T.



4.1 Determine:

4.1.1 The value a. (2)

4.1.2 The equation of g in the form $y = \ldots$ (2)

4.2 If $h(x) = \log_2 x$, describe the translation from f to h. (2)

4.3 Determine:

4.3.1 The value(s) of x for which $f(x).g(x) \leq 0$. (2)

4.3.2 The range of $g(x) - 2$. (2)

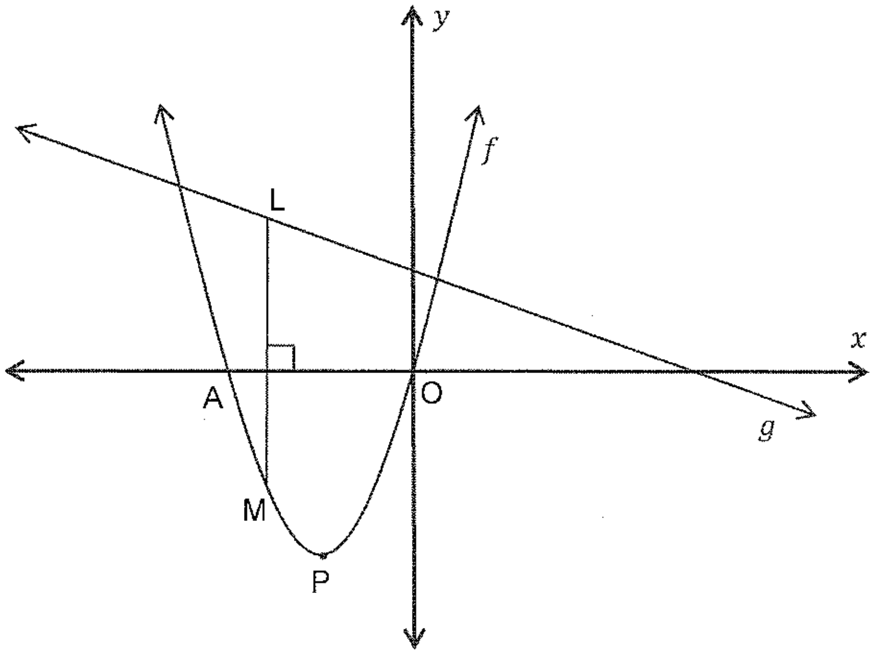
4.4 If p is the equation of $f^{-1}(x)$ that shifted one unit downwards and two units to the left, give the equation of p in the form $p(x) =$ (2)

4.5 Calculate the value of x if $f(x) \geq 2$ (4)

[16]

QUESTION 5

The sketch shows the graph of $f(x) = x(x+3)$ and $g(x) = -\frac{1}{2}x + 2$.



5.1 Determine the coordinates of A. (1)

5.2 Calculate the coordinates of P, the turning point of f. (3)

5.3 Determine the average gradient of f between $x = -5$ and $x = -3$. (3)

5.4 Determine the value(s) of x for which $f(x) > 0$. (2)

5.5 Determine the coordinates of the turning point of h if $h(x) = f(x-2)$. (2)

5.6 L is a point on the straight line and M is a point on the parabola. LM is perpendicular to the x-axis. Show that the length LM can be written as:

$$\text{LM} = -\left(x + \frac{7}{4}\right)^2 + \frac{81}{16}$$

(4)

[15]

QUESTION 6

Given: $h(x) = 2^{-x}$

6.1 Draw a neat sketch of h. (3)

6.2 Determine the equation of q, the graph obtained by reflecting h in the line $y = 0$. (1)

6.3 Write down the equation of h^{-1}, the inverse of h, in the form $y = \ldots$ (2)

6.4 Write down the range of h. (1)

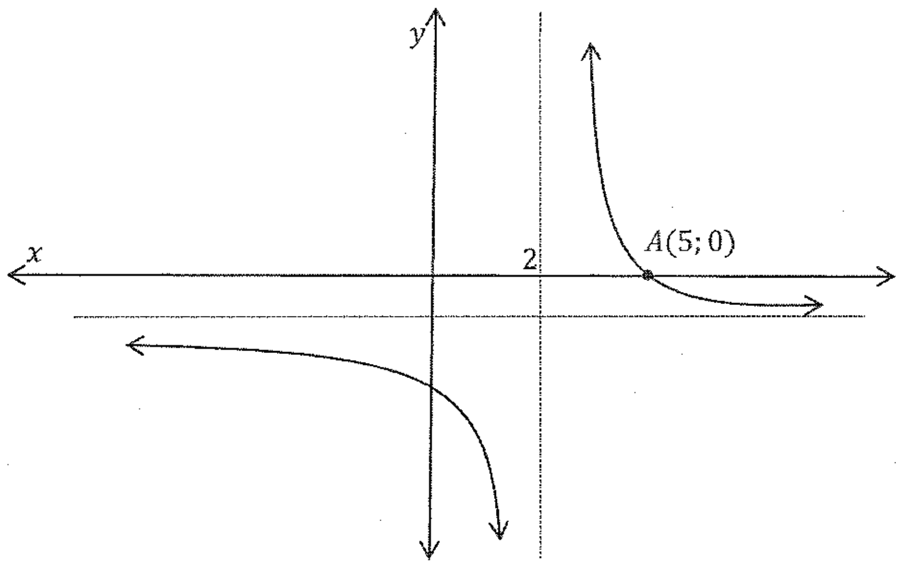
6.5 Sketch the graph of h^{-1} on the same set of axes as h. (2)

6.6 Determine the x-value(s) for which $h^{-1}(x) \geq -3$. (2)

[11]

QUESTION 7

A sketch of the hyperbola $f(x) = \dfrac{d-x}{x-p}$, where $\boldsymbol{p}$ and $\boldsymbol{d}$ are constants, is given below.
The dotted lines are the asymptotes. The point $A(5;0)$ is given on the graph of f.



7.1 Determine the values of $\boldsymbol{d}$ and $\boldsymbol{p}$. (2)

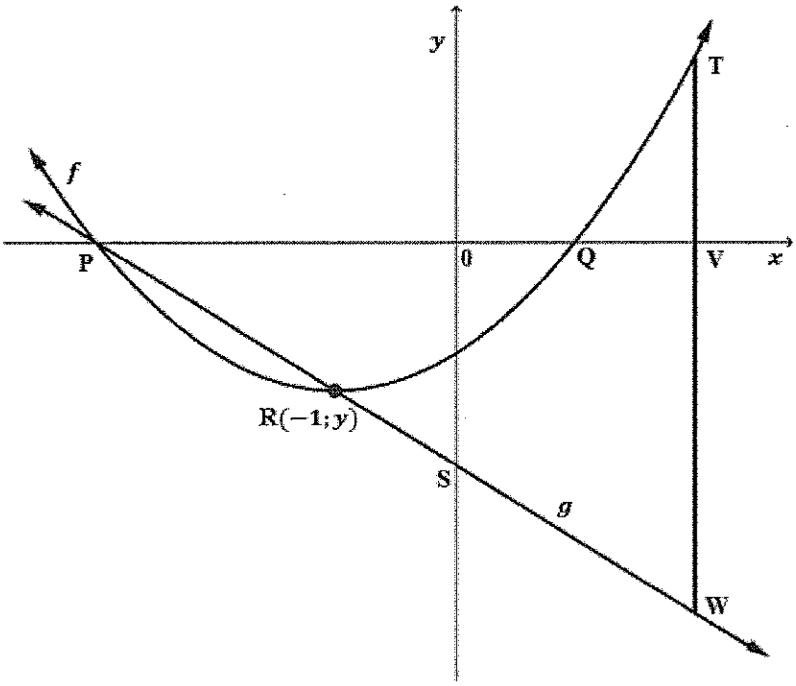
7.2 Show that the equation can be written as $y = \dfrac{3}{x-2} - 1$ (2)

7.3 Write down the image of A if A is reflected about the axis of symmetry $y = x - 3$. (2)

[6]

QUESTION 5

The sketch below shows the graphs of $f(x) = ax^2 + bx - 3$ and $g(x) = -2x - 6$. P and Q are the x-intercepts of $f(x)$. $R(-1;y)$ is the turning point of $f(x)$. The straight line g, passing through P and R, cuts the y-axis at S.



5.1 Determine the coordinates of P. (3)

5.2 Show, by calculation, that the equation of $f(x) = x^2 + 2x - 3$. (3)

5.3 Calculate the length of TW if OV = 2 units. (3)

5.4 The tangent to the graph of f at a point M is parallel to $g(x)$.

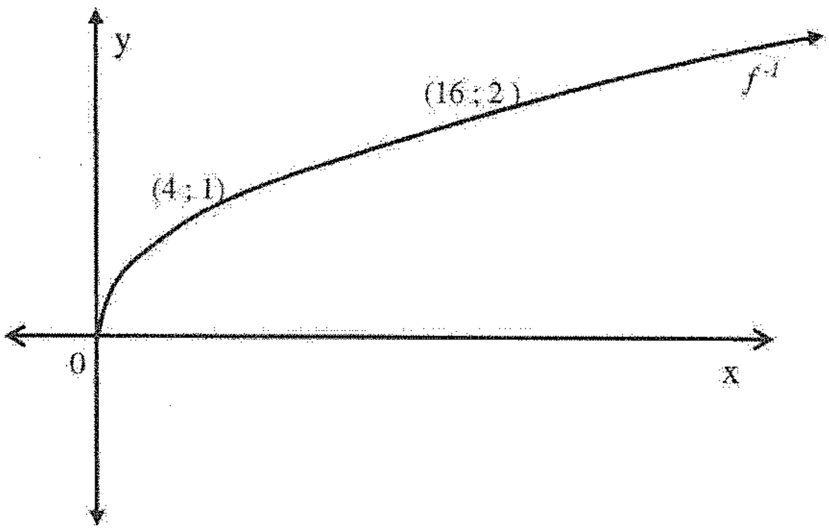
5.4.1 Determine the coordinates of M, the point of contact. (4)

5.4.2 Determine the value of k if $h(x) = g(x) + k$ is a tangent to the graph of f. (2)

[15]

QUESTION 6

The graph of the inverse of $f(x) = ax^2, x \geq 0$, is drawn below. (4; 1) and (16 ; 2) are points on $f^{-1}(x)$.



Determine:

6.1 The value of a. (2)

6.2 The equation of f^{-1}. (2)

6.3 x, if $f(x+2) = 16$. (2)
[6]

QUESTION 7

7.1 John invests R12 500 for p years at a compound interest rate of 9% p.a., compounded quarterly. At the end of the p years his investment is worth R30 440. Calculate the value of p.

7.2 Susan bought a house for R500 000 on an agreement in which she will repay it in monthly instalments at the end of each month for 5 years. Interest is charged at 18% p.a., compounded monthly.

7.2.1 Calculate the annual effective interest rate of the loan. (3)

7.2.2 Calculate Susan's monthly instalments. (4)

7.2.3 Susan decided to pay R12 700 each month as her repayment. Calculate the outstanding balance of the loan after 2 years. (3)

7.2.4 At the end of 2 years, the market value of Susan's house had reduced to R304 200. Determine the annual interest rate of depreciation on the reducing balance value. (3)
[17]

QUESTION 6

Given: $f(x)=\left(\frac{1}{5}\right)^x$

6.1 Determine an equation for f^{-1} in the form $y=$ .... (1)

6.2 Sketch the graphs of f and f^{-1} on the same system of axes on the diagram sheet provided. Clearly show all intercepts with the axes. (4)

6.3 Write down the domain of f^{-1}. (2)

6.4 For which values of x will $f(x).f^{-1}(x)\geq 0$. (2)

6.5 Write down the range of $g(x)$ if $g(x)=-f(x)-3$. (2)

[11]

QUESTION 7

7.1 Hanno's car is currently worth R30 000. Calculate value of the car after 7 years if the value of the car decreases, on the reduced balance method, at the rate 9 percent compounded yearly. (2)

7.2 Monique needs R100 000 in five years' time for her university studies. She decides to get a part-time job to save this amount.
How much should she deposit monthly in an account that earns 10.05 % interest p.a. compounded monthly. Her first payment begins immediately (at the beginning of the first month) and her last payment is at the end of the last month. (4)

7.3 When Mr Rhola retires, he will have R2 500 000 available as a life annuity.
Calculate how long he will be able to withdraw R20 000 a month from this fund if it earns interest at 8% p.a., compounded monthly. (4)

7.4 A loan must be repaid by means of 70 monthly instalments of R2000 each, starting one month after the loan is granted. Interest is fixed at 18% per annum, compounded monthly. Determine the outstanding balance of the loan after 5 years. (3)

[13]

QUESTION 5

5.1 Sarah bought a new laptop. It depreciates in value from R12 000 to R7 500 over a period of three years. Calculate the rate of depreciation per year, if depreciation is on a reducing balance. (3)

5.2 Rueben wants to buy a house costing R1 200 000. He is required to pay a 15% deposit and will borrow the rest from the bank. The bank charges interest at 9,2% per annum, compounded monthly.

5.2.1 How much money will he borrow from the bank? (2)

5.2.2 Calculate his monthly instalment if he needs to pay back the loan over a period of 20 years. (4)

5.2.3 Rueben decides that he can afford to pay R12 000 per month. How long will it take him to repay the loan? Give the answer to the nearest year. (5)

[14]

QUESTION 6

6.1 Given: $f(x)=\frac{-x^2}{2}-3$

Determine $f'(x)$ from FIRST PRINCIPLES. (5)

6.2 Determine:

6.2.1 $D_x\left[\left(x^2-2\right)\left(\frac{1}{x^2}+3\right)\right]$ (4)

6.2.2 $\frac{dy}{dx}$ if $y=4\sqrt{x}-\frac{8}{\sqrt{8}}+\pi x^3$ (3)

[12]

QUESTION 5

Given: $f(x) = \frac{-3}{x+1} + 5$

5.1 Write down the

5.1.1 asymptotes of f (2)

5.1.2 domain of f (2)

5.2 Calculate the intercepts of the graph of f with the axes. (4)

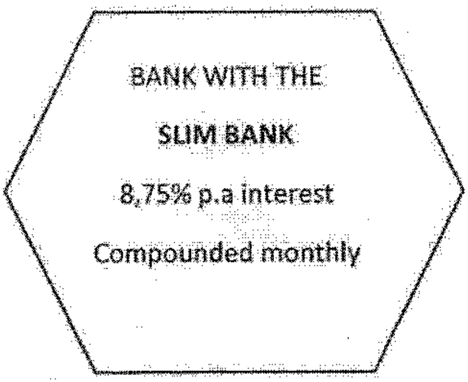
5.3 Sketch the graph of f, for $x > -1$ in your ANSWER BOOK, clearly showing the asymptotes and the intercepts with the axes. (3)

5.4 The graph of f is shifted three units to the right and then reflected about the x - axis to form the graph of h. Write down the equation of h. (3)
(Note that the restrictions in QUESTION 5.4 do not apply to this question).

[14]

QUESTION 6

Read the advertisement carefully. The questions below refer to the advertisement.



6.1 You decide to invest R1 000 into the account as advertised above.
How many months will it take the investment to double? (4)

6.2 John purchased a new car. The bank offered him a loan at an effective interest rate of 16,4% p.a. Determine the nominal interest rate, compounded monthly, that he is required to pay. (3)

6.3 A farmer sets up a sinking fund. He plans to accumulate R2,3 million in the fund at the end of 8 years, at 12% p.a. interest compounded quarterly. Calculate the quarterly payments into the fund if his first payment is made in 3 months' time. (4)

6.4 Samuel plans to attend the 2018 Soccer World Cup in Russia. He wants to book an all-inclusive soccer-package through a travel agency at the price of R135 000. Samuel takes a personal loan from a bank. He has the option of taking out a loan of 14,75% p.a, interest compounded monthly. He then decides to take the loan and chooses to pay it back in 18 equal monthly instalments, starting three months after receiving the loan.

6.4.1 Determine the total amount of the loan when he started repaying it. (2)

6.4.2 Determine his monthly instalment. (4)

[17]

QUESTION 7

7.1 If $f(x) = 5 - 2x^2$, determine $f^{/}(x)$ using FIRST PRINCIPLES. (5)

7.2 Determine:

7.2.1 $Dx\left[x^4 - 2x + \frac{1}{x^2}\right]$ (4)

7.2.2 $\frac{dy}{dx}$, if $y = \frac{2x-3}{\sqrt[4]{x}}$ (4)

[13]

QUESTION 8

8.1 Given: $f(x) = -\frac{3}{x}$

Determine the derivative of $f(x)$ from first principles. (4)

8.2 If $f(x) = (x+3)^2$, determine:

8.2.1 $f'(x)$ (3)

8.2.2 Determine the gradient of the tangent to $f(x)$ at $x = 3$ (2)

8.3 Determine:

$$D_x[6\sqrt[5]{x^2} + \frac{4}{x} - \pi x^3]$$ (4)

8.4 It is given that $f(x) = ax^3 - 24x + b$ has a local minimum at (-2; 17).

Calculate the values of a and b. (4)

[17]

QUESTION 8

8.1 Given: $f(x) = -2x^2 + p$

Determine $f'(x)$ from first principles. (5)

8.2 Determine: $D_x\left[4\sqrt[3]{x} + \frac{1}{3x} + 2\right]$ (4)

[9]

QUESTION 9

Given: $f(x) = (x-1)^2(x+3)$

9.1 Determine the turning points of f. (5)

9.2 Draw a neat sketch of f showing all intercepts with the axes as well as the turning points. (4)

9.3 Determine the coordinates of the point where the concavity of f changes. (3)

9.4 Determine the value(s) of k, for which $f(x) = k$ has three distinct roots. (2)

9.5 Determine the equation of the tangent to f that is parallel to the line $y = -5x$ if $x < 0$. (6)

[20]

QUESTION 10

A closed rectangular box, with a rectangle as base, has a length $(2x)$ cm and width (x) cm. The total surface area (all 6 sides) is 243 cm^2.

10.1 Show that the height, h, is equal to $\left(\frac{81}{2x} - \frac{2x}{3}\right)cm$. (2)

10.2 Show that the volume of the box, in terms of x, is given by the formula:

$V = 81x - \frac{4}{3}x^3$ (2)

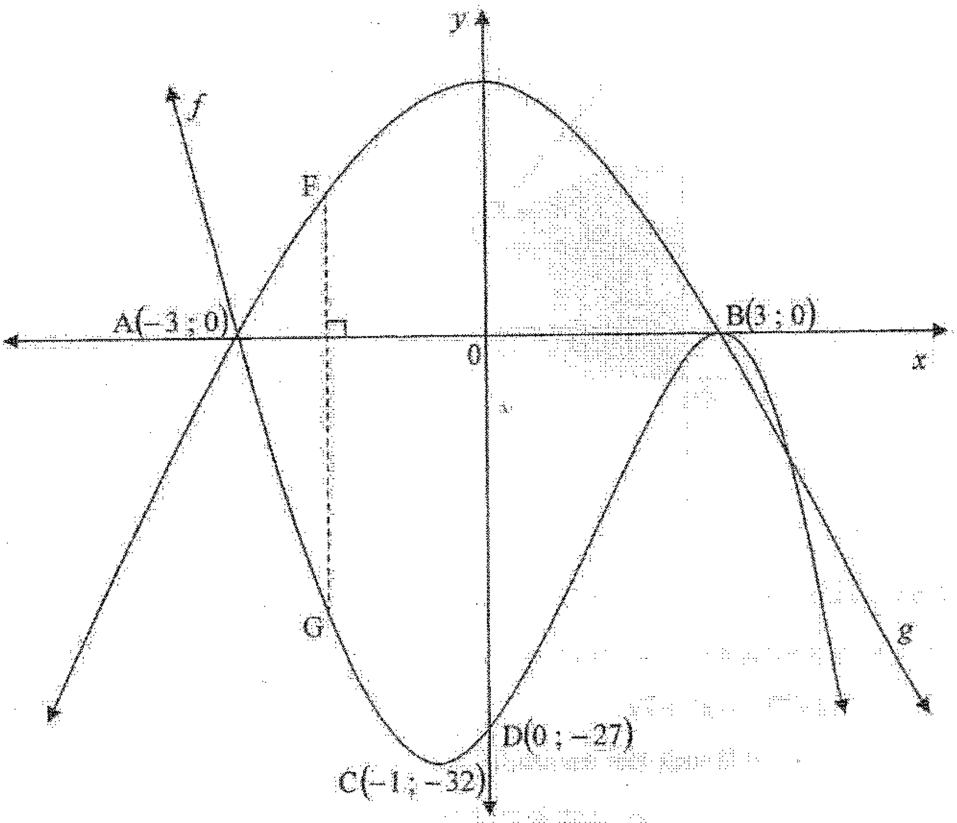
10.3 Calculate the value of x if the volume of the box is a maximum. (3)

[7]

QUESTION 8

The graph of $f(x) = -x^3 + 3x^2 + 9x - 27$ and $g(x) = -x^2 + 9$ are sketched below.

$A(-3\,;0)$ and $B(3\,;0)$ are the $x-$intercepts of both f and g. $C(-1;-32)$ and $D(0\,;-27)$ are the local minimum and y-intercept of f respectively.



8.1 Use the given graphs to determine the values of x for each of the following conditions.

8.1.1 f is a strictly increasing function. (2)

8.1.2 $f(x).g(x) < 0$ (3)

8.1.3 $f'(x)$ and g are both negative (3)

8.2 Determine the equation to f at the point of inflection in the form $y = mx + c$ (5)

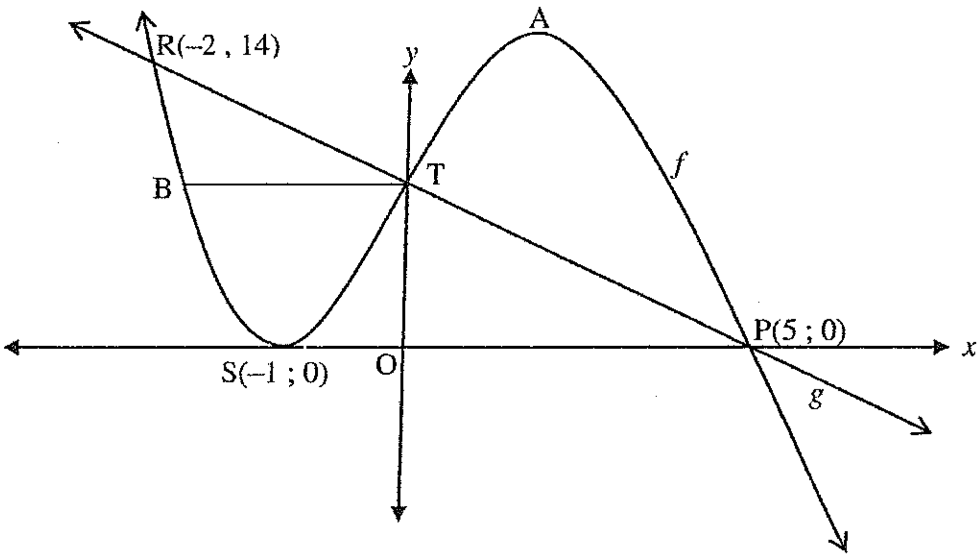
8.3 For which values of x will the graph of f be concave down ? (2)

8.4 FG is a line perpendicular to the x-axis between A and B, where F and G are points on g and f respectively. Determine the value of x (correct to two decimal places) for maximum length of FG. (5)

[20]

QUESTION 7

In the diagram, the graph of $f(x) = -2x^3 + 6x^2 + 18x + 10$ and $g(x) = -2x + 10$ are drawn $P(5\ ;\ 0)$ and $S(-1\ ;\ 0)$ are points on the x- axis and T is the y- intercept of f and g. S and A are turning points of f. f and g intersect at $R(-2\ ;\ 14)$



7.1 Calculate the coordinates of A. (5)

7.2 Determine the equation of the tangent to f where $x = 2$ in the form $y = mx + c$. (5)

7.3 For which value(s) of x will:

7.3.1 f be concave down? (3)

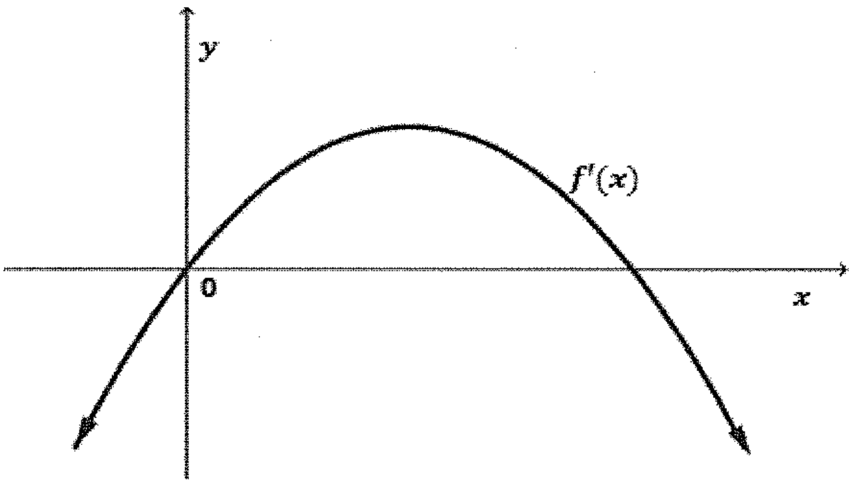
7.3.2 $x \cdot f'(x) > 0$ (2)

7.4 BT is parallel to the x – axis. Prove that the area of ΔRBT is less than 4 square units. (4)

[19]

QUESTION 9

9.1 For a certain function $f(x)$, the first derivative is given as $-3x^2+6x$



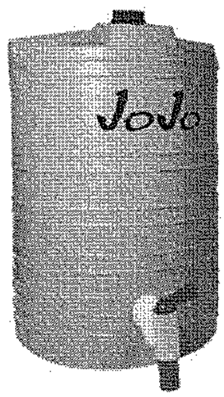
9.1.1 Calculate the $x-$ coordinates of the stationary points of $f(x)$. (3)

9.1.2 Determine the value of x at the point where the concavity of $f(x)$ changes. (1)

9.1.3 It is further given that $f(x)=-ax^3+bx^2+cx-4$ and $f(2)=0$, draw a rough sketch of $f(x)$, clearly showing the coordinates of the turning points. (3)

9.2 A farmer wanted to determine the amount of water he used every week during the drought. He measured the amount of water drawn from a Jojo water tank used on the farm. He determined that the volume of water, in litres, t weeks after he started measuring was:

$$V=-100t^2+200t+2400$$



9.2.1 After how many weeks was the volume at a maximum? (3)

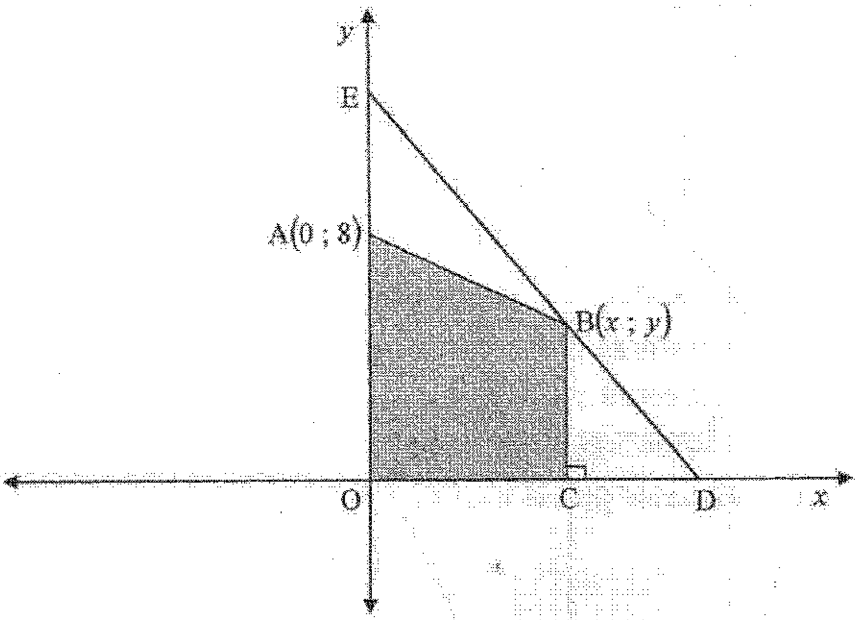
9.2.2 After how many weeks will the tank be empty? (3)

9.2.3 Determine the rate at which the volume changes with respect to the time at $t=3$. (2)

[15]

QUESTION 9

The equation of ED is $y+2x=10$, where x and y are always positive, as shown below.



B$(x\,;\,y)$ is any point on ED. A is the point $(0\,;\,8)$, and BC is perpendicular to the x – axis

9.1 Determine an expression for BC in terms of x. (2)

9.2 Show that the area AOCB $= -x^2 + 9x$ (2)

9.3 Calculate the coordinates of B such that the shaded area AOBC is a maximum. (3)

[7]

QUESTION 10

10.1 Peter plays hockey and soccer at school. He has a 40% chance of being selected for the hockey team and a 60% chance of being selected for the soccer team and a 30% chance of being selected for both teams.

Calculate the probability that Peter will be selected for the hockey or soccer team (3)

10.2 A survey was conducted asking 60 people with which hand they write and what colour hair they have.

The results are summarized in the table below:

		HAND USED TO WRITE WITH		
		Right	**Left**	**Total**
HAIR COLOUR	**Light**	a	b	20
	Dark	c	d	40
	Total	48	12	60

The survey concluded that the "hand used for writing" and "hair colour" are independent events.

Calculate the values of a, b, and c. (5)

[8]

QUESTION 11

11.1 The digits 0 to 9 are used to form codes.

11.1.1 Determine the number of unique 6-digits codes that can be formed if repetition of digits is allowed. (1)

11.1.2 Determine the probability of unique 6-digit codes that can be formed that start with a 9 and end with a 2, if repetition of digits is not allowed. (3)

11.2 The digits 0 to 9 are used to form 10-digit codes.

Determine the number of unique 10-digit codes that can be formed if the 2 and the 3 may not appear next to each other and repetition of digits is not allowed. (4)

[8]

TOTAL: 150

QUESTION 8

A factory produces bicycles. The profit of the factory in Rand per week, P, is dependent on the number of workers, x, according to the formula:

$$P = -x^3 + 75x$$

Determine:

8.1 The profit if seven workers work for a week. (2)

8.2 The number of workers that must be employed to ensure maximum profit per week. (4)

8.3 The maximum weekly profit. (2)

[8]

QUESTION 9

9.1 The probability that it will rain on a given day is 52%. A car accident has a 7% chance of happening in dry weather and four times as likely to happen in wet weather.
What is the probability that an accident will happen on any given day? (5)

9.2 Consider the word **PURPLE.**
How many different ' words' can be formed from these letters? (3)

9.3 Mr Shaw has seven different shirts and four different trousers hanging in his closet.
Calculate:

9.3.1 How many different ways can the clothes be hanged, if all the shirts and all the trousers must hang together. (3)

9.3.2 The probability that one shirt will hang at the beginning of the rail and one pair of trousers at the end of the rail. (4)

[15]

TOTAL: 150

QUESTION 10

10.1 Given $P(A) = 0{,}45$, $P(B) = 0{,}3$ and $P(A or B) = 0{,}615$.

Are the events A and B: (Show all calculations)

10.1.1 Mutually exclusive? (3)

10.1.2 Independent? (3)

10.2 Three boys and five girls are going for a photoshoot.They are all seated next to each other in the same row.

10.2.1 In how many different ways can the entire group be seated? (1)

10.2.2 One boy and girl are a couple and want to sit next to each other. In how many different ways can the entire group now be seated? (2)

10.3 Consider the word BELLVILLE. The letters of this word are randomly arranged to form new words.

Calculate the probability that the word will start with a 'L'. (3)

[12]

TOTAL : 150

QUESTION 11

11.1 You have to select a new password for your "Dropbox" account on your computer. The password must consist of 3 digits and 2 letters of the alphabet in that order. The digit 0(zero) is not allowed and no consonants are allowed. Any digit may be repeated but the vowels may not be repeated. How many different passwords are possible? (3)

11.2 Using the letters in the word FUNDAMENTALS , determine:

11.2.1 The number of unique 12 letter arrangements that can be formed (3)

11.2.2 The probability that a new arrangement will start and end with the letter **N** (3)

11.3 In a city where the ratio of male to female is 1 : 2, one person is selected to flip a fair coin for the kick-off of a soccer tournament.

11.3.1 Draw a tree diagram to show all possible outcomes of the coin toss. (3)

11.3.2 Determine the probability that it will be a woman who flips the coin. (1)

11.3.3 Determine the probability that it will be a man who flips a head. (2)

[15]

TOTAL: 150